Towards Enhancing Conceptual Knowledge in Algebra through Diagrammatic Self-explanation in an Intelligent Tutoring System

Tomohiro Nagashima
Human-Computer Interaction Institute, Carnegie Mellon University
tnagashi@cs.cmu.edu

ABSTRACT: In mathematics education, it is still unclear how instruction can support learning conceptual knowledge, procedural knowledge, and connections between them. Particularly, research suggests that it is harder to acquire conceptual knowledge than procedural knowledge in algebra. Tape diagrams, a representation used to visualize a relationship between quantities in an equation, have been studied to explore their potential benefits in supporting conceptual knowledge; however, their effectiveness is still not entirely clear especially for low-ability students. To effectively foster students' conceptual knowledge in algebra, we propose a novel instructional approach integrating self-explanation into the use of tape diagrams in an intelligent tutoring system. The proposed study will design and test this approach, called diagrammatic self-explanation, where students manipulate tape diagrams as a way of self-explanation. The study will make contributions to the fields of learning analytics and the learning sciences by 1) establishing an effective instructional strategy using the combination of tape diagrams and self-explanation and 2) designing an adaptive tutor for equation solving with tape diagrams.

Keywords: Conceptual Knowledge, Tape Diagrams, Self-explanation, Equation Solving, Algebra, Intelligent Tutoring Systems.

1 INTRODUCTION

1.1 Conceptual Knowledge in Mathematics

One of the biggest challenges in mathematics education is how to support learning conceptual knowledge (CK), procedural knowledge (PK), and connections between them (Schneider, Rittle-Johnson, & Star, 2011). A widely-accepted view is that the development of CK and PK is interactive, where the development of one type of knowledge leads to the other and vice versa (Rittle-Johnson & Schneider, 2014). Yet, it has been found more difficult to gain CK compared to PK in some mathematics fields, including algebra (Matthews & Rittle-Johnson, 2009). Despite the importance of fostering CK, however, current teaching practices are too often focused on teaching procedures without offering explanations on conceptual understanding of such procedures (National Council of Teachers of Mathematics, 2014).

Conceptual knowledge is a complicated notion for which researchers have adopted a variety of definitions. Crooks and Alibali (2014) suggest using two types of definitions: "general principle knowledge" and "knowledge of principles underlying procedures" (p. 366). The former refers to the fundamental and general knowledge about the domain, such as rules and definitions whereas the latter involves “knowing why certain procedures work for certain problems and knowing the purpose of each step in a procedure” (Crooks & Alibali, 2014, p.367). We will adopt these definitions of CK in the proposed study.

1.2 Tape Diagrams

One promising way of fostering CK in mathematics instruction is the use of diagrams. Diagrams and other types of external representations have been studied extensively in mathematics education with their effects generally proven to be positive (Mayer, 2005). In algebra, one type of diagram that is thought to be helpful for students is tape diagrams (Murata, 2008). Tape diagrams visually depict relationships among quantities in an equation problem (Figure 1). They are consistently used in mathematics instruction in Japan and Singapore, two of the countries where students perform far better than the world’s average on international mathematics tests (Murata, 2008). In the United States, studies have empirically demonstrated the benefits of the presence of tape diagrams on learning among middle school students (Booth & Koedinger, 2012; Chu, Rittle-Johnson, & Fyfe, 2017; Koedinger & Terao, 2002). However, it is also suggested that prior knowledge may mediate the benefits: several studies indicate that low-ability students were incapable of translating their conceptual understanding of the quantitative relationship across different representations (e.g. tape diagrams and algebraic equations, tape diagrams and word problems) (Booth & Koedinger, 2012; Chu, Rittle-Johnson, & Fyfe, 2017). This implies the need for more careful design and additional scaffolds to support not only high-ability students but also low-ability students.

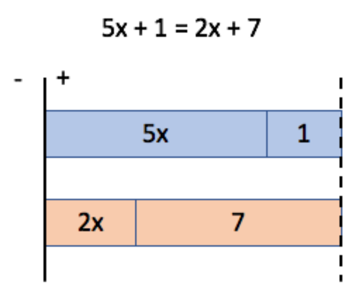

Figure 1: An example of a tape diagram representation. Tape diagrams visualize the relationship between the quantities in the equation.

1.3 Self-explanation

Another potential approach to boosting CK is self-explanation. Self-explanation is an instructional strategy in which students generate explanations in order to understand newly-introduced information by connecting it with their prior knowledge (Rittle-Johnson, Loehr, & Durkin, 2017; Wylie & Chi, 2014). It has consistently been shown effective in a variety of domains both in a paper-and-pencil format and in a computer environment (Wylie & Chi, 2014). In mathematics education, research has illustrated that self-explanation is effective in promoting CK and PK (Rittle-Johnson et al., 2017), but it has also been found that the effectiveness of self-explanation in promoting CK is limited in a classroom context (Rittle-Johnson et al, 2017). In an effort to facilitate robust learning in an authentic context, however, researchers have studied and demonstrated that intelligent tutors can effectively enhance students' CK, particularly through scaffolding self-explanations via providing a list of possible responses or self-explaining in a fill-in-a-blank form (e.g. Rau, Aleven, & Rummel, 2015). This suggests that intelligent tutoring software, when designed appropriately, can meaningfully foster CK through self-explanation prompts. As self-explanation can potentially support students' understanding of tape diagrams and to help them connect different representations, it is worthwhile to explore the potential of integrating self-explanation into tape diagram use.

The proposed study will make contributions to the fields of the learning sciences and learning analytics by 1) establishing an effective instructional strategy involving self-explanation in equation solving with tape diagrams and by 2) designing and evaluating an adaptive tape diagram tutor which adapts scaffold/prompt types and levels to students' prior knowledge.

2 PROPOSED STUDY AND RESEARCH QUESTIONS

Towards developing an effective instructional strategy for enhancing CK, we propose a novel approach of integrating self-explanation into the use of tape diagrams for equation solving. In doing so, we will use tape diagrams not as an additional representation to algebraic equations or word problems, but rather as an active constructive activity which we call diagrammatic self-explanation, where students are asked to manipulate tape diagrams when solving an equation. We will develop diagrammatic self-explanation activities in Lynnette, a web-based intelligent tutor for equation solving that supports practices across problems of varying difficulty (Long & Aleven, 2017). As is common in Intelligent Tutoring Systems (ITS), Lynnette offers step-by-step personalized guidance based on students' inputs, and personalized problem selection.

In diagrammatic self-explanation, it is hypothesized that manipulating tape diagrams can help develop CK, such as the concept of mathematical equivalence (Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012) and variables, through visualizing quantitative relationships. Figure 2 illustrates an example of diagrammatic self-explanation. In this example, students are asked to manipulate the tape diagram following the transformation steps shown on the left and identify an error.

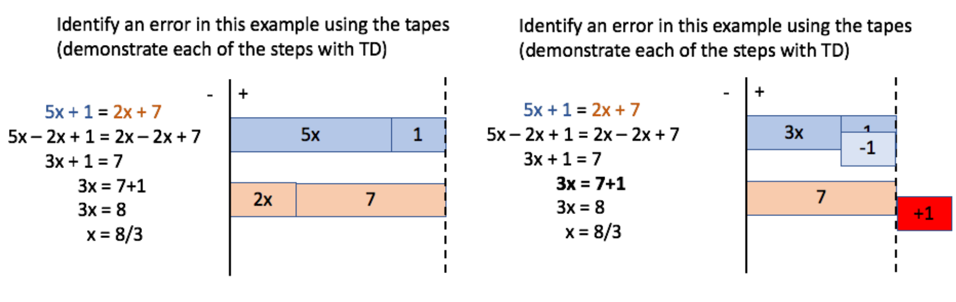

Figure 2: Diagrammatic self-explanation with an incorrect example. The tape diagrams on the left show 5x + 1 = 2x + 7 (original equation) and the diagrams on the right show 3x = 7 + 1 (intermediate step) .

To the best of our knowledge, the instructional potential of "manipulable" tape diagrams in equation solving has not been investigated to date, presumably due to the difficulty of manipulating tapes on paper. Also, in equation solving (e.g. Looi & Lim, 2009), tape diagrams have traditionally been used in the problem representation phase (e.g. constructing tape diagrams given a word problem) but not in the problem solution phase (Mayer, 1985). As constructive activities such as sketching and drawing have been shown effective for learning (e.g. Wu & Rau, 2018), we believe that diagrammatic self-explanation, where students manipulate tape diagrams in the problem solution phase, can be beneficial to students. Our proposed study will address the following research questions:

- *Will diagrammatic self-explanation enhance students' CK?*
- *Will diagrammatic self-explanation enhance low-ability students' CK?*

3 RESEARCH PLAN AND CURRENT STATUS

3.1 User-centered Investigation on the Use of Tape Diagrams

Despite their popularity and success in Asian countries, tape diagrams are not necessarily a familiar representation in other countries, including the United States (Murata, 2008). In order to explore the design, potential instructional impacts, and any difficulties associated with tape diagrams, we will first conduct qualitative user research with teachers and students in middle schools in the US. Specifically, we will conduct interviews and task analysis, followed by iterative prototyping co-designing with teachers and testing of the prototypes in Lynnette with students. The findings would inform the instructional design of diagrammatic self-explanation and necessary training for teachers and students, as well as the design of the ITS interface.

3.2 Planned Experimental Study

Once we have perfected our design through prototyping, we will conduct an experiment examining the effectiveness of diagrammatic self-explanation in equation solving.

3.2.1 Participants and Materials

Seventh- and eighth-grade students at schools in the US will participate in the study, which will take place as part of their regular mathematics instruction. Equation solving activities will be prepared in Lynnette. We will also develop pre- and post-assessments on CK in equation solving based on past studies with Lynnette as well as from mathematics education literature.

3.2.2 Study Design

The proposed study will conduct an *in vivo* experiment (i.e. a rigorously controlled experiment in a natural classroom setting) examining whether the diagrammatic self-explanation can improve students' performance on CK. Students will be randomly assigned to either of three conditions. The first condition involves diagrammatic self-explanation as a way of solving algebraic equations. Students in the second condition will solve algebraic equations but tape diagrams will be shown as an additional reference to the algebraic equations, rather than as manipulable tape diagrams. In the third condition, students will solve algebraic equations without tape diagrams.

3.2.3 Procedure

Students will first be asked to work on the pre-test in the ITS. After being assigned to either of our three conditions, they will be asked to solve the equation problems with diagrammatic self-explanation prompts or no prompts. All groups will receive the problems at the same difficulty level and the total amount of time spent will be matched across the conditions. Following that, students will be asked to complete the online post-test.

3.3 Expected Results

We hypothesize that students in the tape diagram conditions (first and second conditions) will perform better than the no tape diagram condition.

Regarding our second research question on individual differences, when we compare the results from the first and second conditions, we expect to see an interaction effect between math ability, assessed by the pre-test, and the type of tape diagram use. Specifically, we expect that diagrammatic self-explanation will help both low-ability and high-ability students while using tape diagrams as a reference (no manipulation) will only be effective for high-ability students.

3.4 Current Status

We have reviewed the literature on related topics and have started the qualitative investigation of the design of the interactive tape diagrams with teachers and students.

4 FUTURE PLAN: ADAPTIVE TAPE DIAGRAM TUTOR

Our proposed study will test whether diagrammatic self-explanation can promote the learning of CK in equation solving. Although what follows might change depending on the results of our first study, we plan to develop an adaptive tape diagram tutor, which would vary the prompt/scaffold type and type of tape diagram representation based on students' level of CK, because it is likely that students' individual differences in prior knowledge influence whether and how much students benefit from the use of tape diagrams.

To explore this approach, we will investigate whether we can use students' interaction data from Lynnette to identify their levels of CK when they work on activities. Once we identify their levels of CK, we would be able to provide different diagrammatic scaffolds or activities to avoid over-scaffolding or under-scaffolding and to meaningfully support students' conceptual understanding in Lynnette (e.g. constructing tape diagrams from a list of possible tape options, selecting a correct tape diagram representation from the list of possible answers). We believe that our first study will provide the foundation for the idea of the adaptive tutor.

REFERENCES

Booth, J. L., & Koedinger, K. R. (2012). Are diagrams always helpful tools? Developmental and individual differences in the effect of presentation format on student problem solving: Development of diagram use. *British Journal of Educational Psychology*, *82*(3), 492–511.

Chu, J., Rittle-Johnson, B., & Fyfe E. R. (2017). Diagrams benefit symbolic problem-solving. *British Journal of Educational Psychology, 87*(2), 273-287.

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, *34*(4), 344–377.

Koedinger, K. R., & Terao, A. (2002). A cognitive task analysis of using pictures to support pre-algebraic reasoning. In *Proceedings of the Annual Meeting of the Cognitive Science Society* (Vol. 24, No. 24).

Long, Y., & Aleven, V. (2017). Educational game and intelligent tutoring system: A classroom study and comparative design analysis. *ACM Transactions on Computer-Human Interaction (TOCHI)*, 24(3).

Looi, C. K., & Lim, K. S. (2009). From bar diagrams to letter-symbolic algebra: a technology-enable bridging. *Journal of Computer Assisted Learning, 25*(4), 358-374.

Matthews, P.G. & Rittle-Johnson, B. (2009). In pursuit of knowledge: Comparing self-explanations, concepts, and procedures as pedagogical tools. *Journal of Experimental Child Psychology, 104*, 1-21.

Matthews, P.G., Rittle-Johnson, B., McEldoon, K., & Taylor, R.T. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *Journal for Research in Mathematics Education, 43*, 316-350

Mayer, R. E. (1985). Mathematical ability. In R. J. Sternberg (Ed.), *Human abilities: An information processing approach* (pp. 127-150). San Francisco: Freeman.

Mayer, R. E. (2005). Cognitive theory of multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning.* New York, NY: Cambridge University Press.

Murata, A. (2008). Mathematics teaching and learning as a mediating process: the case of tape diagrams. *Mathematical Thinking and Learning*, *10*(4), 374–406.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston VA: Author.

Rau, M. A., Aleven, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology, 107*(1), 30.

Rittle-Johnson, B., Loehr, A. M., & Durkin, K. (2017). Promoting self-explanation to improve mathematics learning: A meta-analysis and instructional design principles. *ZDM*, *49*(4), 599–611.

Rittle-Johnson, B., & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. *Oxford handbook of numerical cognition*, 1102-1118.

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology*, *47*(6), 1525–1538.

Wu, S. P. W., & Rau, M. A. (2018). Effectiveness and efficiency of adding drawing prompts to an interactive educational technology when learning with visual representations. *Learning and Instruction, 55*, 93-104.

Wylie, R., & Chi, M. T. H. (2014). The self-explanation principle in multimedia learning. In R. E. Mayer (Ed.), The Cambridge Handbook of Multimedia Learning (2nd Ed., pp. 413-4320. Cambridge University Press.